DSBN DISTRICT SCHOOL BOARD OF NIAGARA
Mathematics Newsletter

measurement

One of the first spatial concepts we develop is the notion of length. We notice how tall people are, how long their hair is, who gets the longer piece of licorice, which toy car goes farther… When we compare two lengths of string, what is it that we are paying attention to? We might answer, "Length!" However, what *IS* "length"? We know that you cannot define something using the word you are defining. Here is the dilemma: there is no word in English. So, because there isn't, we should invent one. We are paying attention to the "long-ness" or "short-ness" of the string.

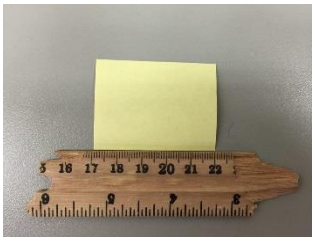
We have the same dilemma when we try to define area. Yes, we're quantifying the space it takes up, but what attribute are we considering? Again, there is no word in English. The invented word "covering-ness" works really well.

This is more than a silly conversation or mere semantics. Van de Walle states the first Big Idea of Measurement: *Measurement involves a comparison of an attribute of an item or situation with a unit that has the same attribute. Lengths are compared to units of length, area to units of area, time to units of time, and so on.* (K-2 pg. 269, 3-5 pg. 312, 6-8 pg. 298) Helping students to understand the attribute of length and area warrants our explicit attention.

All measurement is actually a comparison. The object being measured is being compared with a unit. A 30-cm book is measured by placing a unit of 1 centimetre down and counting how many units will match the length of the book. This process, called ***iteration***, is also of critical importance in fractions – for example, how many fourths are in 3 ½? (Van de Walle, Grades 6-8, p. 300) A ruler, or any measurement tool, simply eliminates the tedium of lining those units up without overlapping or leaving spaces.

Formative Assessment Note …from Van de Walle

How can I measure the object in the picture with my broken ruler?



Van de Walle
K-2 pg. 272
3-5 pg. 315
6-8 pg. 302

Why are we doing AREA twice in the Scope and Sequence / Long Range Plan?

The December Learning Cycle will deal with the linear aspects of measurement and its intentional connections that give context for:

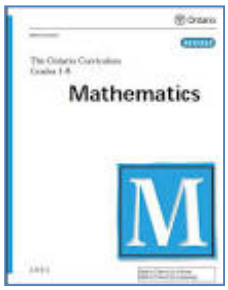
- the linear model for addition and subtraction (as in a number line oriented either horizontally or vertically, as with a thermometer). **Research has shown that the linear model is a powerful and effective tool for thinking and communicating about addition and subtraction for small numbers, large numbers, fractions, decimals, *and is the foundation of work in the intermediate grades with integers.***
- establishing the relationship between the side lengths of a rectangle and its area
- using area measure contexts to build the connection of the array/area model to multiplication

In this way, we are intentionally looking not only at measurement expectations, but also expectations in Number Sense and Numeration. We hope you will intentionally:

- pose measurement problems that have students adding and subtracting
- use numbers appropriate for your grade level to give students additional opportunities to build their sense of number and quantity from a linear/measurement perspective.
- repeating linear measures to evoke skip-counting on a number line, a foundation for multiplication
- encourage students use a linear model to justify their thinking where it makes sense to do so

When we revisit area in February, we will intentionally continue to make connections to the array and multiplication, and use our understanding of area to build the understanding of volume and capacity.

During the November/December Learning Cycle for Measurement, Grade 6 teachers should focus on the area of squares and rectangles. The area connection from rectangles to parallelograms and triangles will be developed in the second term **through investigation**.



Delving into the Mathematics Curriculum…

Many measurement expectations use the following phrase:

Determine, through investigation using a variety of tools and strategies…

1m42 – describe, through investigation using concrete materials, the relationship between the size of a unit and the number of units needed to measure length
2m41 – describe, through investigation, the relationship between the size of a unit of area and the number of units needed to cover a surface
5m41 – determine, through investigation using a variety of tools and strategies (e.g., building arrays), the relationships between the length and width of a rectangle and its area and perimeter, and generalize to develop the formulas [i.e., Area = length x width; Perimeter = (2 x length) + (2 x width)]
8m38 – determine, through investigation using concrete materials, the surface area of a cylinder

Every grade has expectations with this wording.

What is the purpose of expectations that are worded "through investigation…"?

- Developing mathematical ideas through investigation gives students opportunity for:
  - the development of independent mathematical thinking
  - engagement in mathematical processes such as:
    - organizing
    - recording data
    - pattern searching
    - conjecturing
    - inferring
    - justifying
    - explaining conjectures and generalizations.

  It is these thinking processes which enable an individual to learn more mathematics, apply mathematics in other discipline and in everyday situations and to solve mathematical (and non-mathematical) problems.

  http://math4teaching.com/2010/03/09/what-is-mathematical-investigation/

- By resorting too early to a 'purified', strictly deductive version of a certain mathematical domain, one runs the risk of stifling the student's personal mathematical reasoning instead of developing it. (Barbin, 1994)

Jo Boaler reports to us that the brain is not learning as it cranks out correct answers; it is learning when we make errors, re-think, reason, conjecture and test theories, look for patterns and come up with generalizations that help us to do more mathematical reasoning. Engagement in these thinking processes means students are actively engaged in complex cognitive processes. This is compared to a more traditional approach in which teachers told students formulas and had students spend time plugging in different kinds of numbers to get right (or incorrect) answers. Leaders in Educational Thought: Mathematics Knowledge-12, 2014; http://learnteachlead.ca/projects/leaders-in-educational-thought-mathematics-k-12/

Did you know that Photo Math is an app that allows someone to capture a traditional textbook math question and calculates the answers, giving all of the steps needed to solve correctly? What is the difference in cognitive load between "mucking around with a mathematical idea to see how finding area works" and doing the odd-numbered questions on page 135 that all have to do with calculating area using a formula?
How will 21st Century technologies affect the need to crank out correct answers?

Formative Assessment Note …from Van de Walle

Estimation tasks are a good way to assess students' understanding of both measurement and units. Use a checklist while students estimate measures of real objects inside and outside the classroom. Prompt students to explain how they arrived at their estimates to get a more complete picture of their measurement knowledge. Asking only for a numeric estimate and not asking for an explanation can mask a lack of understanding and will not give you the information you need to provide appropriate remediation. (3-5, p. 322)

Primary Learning: Linear Measurement
Supporting the Number Line

Big Ideas from Van de Walle:

- Measurement involves a comparison of an attribute of an item or situation with a unit that has the same attribute. Lengths are compared to units of length, areas to units of area, time to units of time, and so on.
- Before anything can be measured meaningfully, it is necessary to understand the attribute to be measured.
- Estimation of measures and the development of benchmarks for frequently used units of measure help children increase their familiarity with units, preventing errors in measurements and aiding in the meaningful use of measurement.
- Measurement instruments (e.g., rulers) are tools that replace the need for actual measurement units.

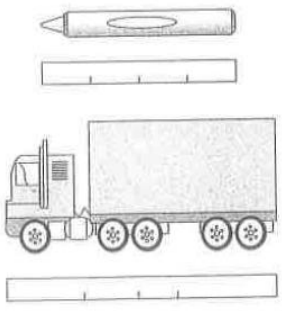
Check out the Guide to Effective Instruction K-3 Measurement for the Characteristics of Learning and Instructional strategies by grade to consider while focusing on Measurement.

Look in Van de Walle Table 15.1 on p. 271 for a Recommended Sequence of Experiences for Measurement Instruction that may help you think about the types of activities you provide for your students.

Possible Activities:

Linear Measurement:
Longer, Shorter, Same, Activity 15.7, VDW p. 280 (Kindergarten, Grade 1) Length (or unit) Hunt. Activity 15.8, VDW p. 280 Crooked Paths, Activity 15.10, VDW p. 281 (possibly use non-standard and standard units)
Estimation:
Strategies/Tips for teaching estimation, VDW p. 277-279 Personal Benchmarks, Activity 15.3 VDW p. 275
Perimeter:
Measuring Indirectly – Activity 4.13 Big Ideas with Dr. Small K-3, p. 97 Learning Connection # 2 – Perimeter, Guide to Effective Instruction K-3 Measurement, p. 125-126
Area:
Two-Piece Shapes, Activity 15.21, VDW pg. 294 (connection to Geometry!) Fill and Compare, Activity 15.22, VDW pg. 294

Note: Time and temperature can be taught throughout the year by providing daily class experiences



Formative Assessment Note …from Van de Walle

For a quick diagnostic interview, give a student the two diagrams as shown and ask them to tell you the length of each object. Does the student simply count the units for both objects, or does s/he indicate the issue with the different length units in the second diagram? If the student simply counts the units, this is an indication that s/he does not understand that the units must be of equal size. Having the student use a wide variety of different kinds of units may actually be more confusing. Instead, having the student consistently use the same kind of unit in measurement activities may help them to realize that the units need to be equal in length.

(K-2 p. 282)

Primary Learning: Connecting the Strands
Number Sense and Algebra – Using the Number Line

Addition and Subtraction	Adding and Subtracting Money	Expressions and Equality
Teaching Student-Centered Math - Van de Walle Pre-K-2: Up and Down the Number Line **Activity 9.3** p.136 How Far To My Number? **Activity 12.4** p.217 *Connection to Measurement* – reading a thermometer and changes in temperature, determining perimeter, adding or subtracting/comparing linear measurements Lessons from The Guide to Effective Instruction in Mathematics: Number Sense and Numeration	*Teaching Student-Centered Math - Van de Walle Pre-K-2:* Coin-Number Addition **Activity 15.18** p.290 Making Change **Activity 15.20** p.292 *Formative Assessment Note* p.290 (use as a quick diagnostic!) Allow students to treat the dollars separate from the cents and combine the two sums or differences.	*Teaching Student-Centered Math - Van de Walle Pre-K-2:* True or False **Activity 9.1** p.134 Ten and Then Some **Activity 13.3** p.230 Open Sentences Activity **13.9** p. 235 Consider modelling equality using two colours, one on the top and one on the bottom. Expressions and Equality lessons from The Guide to Effective Instruction in Mathematics: Patterning and Algebra

Guide to Effective Instruction in Mathematics Volume 5: Teaching Basic Facts and Multidigit Compuations

Take a look at…
P. 8-11 – Types of Problems
P. 23-27 – Basic Addition and Subtraction Facts
P. 47-48 – Open Number Line and Reach the Target
P. 69-78 - Appendix 10-2: Instructions for Games and Activities

Food for thought…
Are you giving students opportunities to answer ***different types*** of addition and subtraction questions?
To ensure variety and opportunities to develop conceptual understanding see Table 9.1 in Van de Walle K-2 p.128

Strings and Number Talks

Using 'Strings' on a regular basis in class is a great way to build computational fluency and model the use of the number line. Representing student thinking allows students to better understand the use of tools, such as the number line, and helps them see ways they can explain their thinking. In Van de Walle, refer to page 214 (Figure 12.6) and page 216-217 (Figure 12.7, 12.8) for samples of *Invented Strategies* students might share!

Teaching Tip *Van de Walle p. 230*
Do not connect multiple expressions using equal signs unless they are equal. For example, when adding 6 and 6 and then adding 3 more to that sum, do not write that as 6+6=12+3=15. Doing this incorrectly reinforces the idea that the equal sign means "and the answer is" rather than indicating equivalence.

Junior Learning: Measurement and the Connection to the Number Line Model

Big Ideas – (from Van de Walle)

- Measurement involves a comparison of an attribute of an item or situation with a unit that has the same attribute. Lengths are compared to units of length, areas to units of area, time to units of time, and so on. (Grades 4-8)
- Estimation of measures and the development of benchmarks for frequently used units of measure help students increase their familiarity with units, preventing errors and aiding in the meaningful use of measurement. (Grades 4-8)
- Measurement instruments (e.g., rulers) group multiple units so that you do not have to iterate a single unit multiple times. (Grades 4-6)
- Area and volume formulas provide a method of measuring these attributes with only measures of length. (Grades 5-8)
- Area and perimeter are related. For example, as the shapes of regions change while maintaining the same area, there is an effect on the perimeter. (Grades 6 - 8)

Technology Tip

Explore the relationship between area and perimeter at http://www.mathplayground.com/area_perimeter.html

Formative Assessment Note

Provide students with a supply of toothpicks. Prepare a paper with two dots on diagonally opposite sides of the paper. Have students determine how far apart the dots are in terms of toothpicks. If students know to line up the toothpicks in a straight line between the dots without any significant gaps or overlapping units, and not use any broken toothpicks, then it may be assumed that they understand the process of using units to measure.

Van de Walle 3-5 p. 322

Possible Activities

Perimeter, Length, Area

Measuring Different Attributes of a Bucket, Figure 16.1, VDW pp. 313-314 (Length: height, Grade 4-6) (Area: covering the surface - Grade 6)
This activity can be revisited in M2 where mass and volume/capacity can be addressed.

Van de Walle (Grades 3-5)

"Estimation Scavenger Hunt" Activity 16.5 page 322
Estimating Measures Using Benchmarks and Chunking, Figure 16.3 pp. 320-321
"Personal Benchmarks" Activity 16.2 page 317
"Two-Piece Shapes" Activity 16.7 page 324
"Rectangle Comparison – Square Units" Activity 16.10 page 327
"Fixed Perimeters" Activity 16.11 page 328
"Fixed Areas" Activity 16.12 page 328

The Box Factory, Fosnot (Grade 6)

Using Measures of Capacity

"That's Cool" Activity 16.16 page 333
"Squeeze Play" Activity 16.17 page 334

Temperature Change

Grade 5 Learning Activity: Weather or Not…, A Guide to Effective Instruction in Math, Measurement Grades 4 to 6, pp. 67-88 (for all grades)

Elapsed Time

Ready for the Bell, Activity 16.19, VdW, pp. 338-339

Intermediate Learning: Measurement and the Connection to the Number Line Model

Attributes, Units and Measurement Sense (Grades 7 & 8)	**Area (Grade 7)**	**Surface Area of Right Prisms (Grade 7)**	**Circles (Grade 8) Circumference, Area, Surface Area of Cylinders**
Figure 14.1 Measuring different attributes of a bucket (page 299 - Van de Walle) Activity 14.3 Estimation scavenger hunt (page 304 – Van de Walle) Activity 14.1 Guess the Unit (page 301 – Van de Walle) Several curricular expectations begin with, "Solve problems that require conversion between…". Conversions are required from larger units to smaller units (i.e., kg to g, m^2 to cm^2…) and should be integrated into problem solving contexts.	See pages 311-312 for tips to developing formulas for area TIPS4RM - Unit 4 - Gr 7 Activity 14.9 Cover and Compare (page 310 – Van de Walle) Figure 14.13 Trapezoids (page 314 – Van de Walle) Dan Meyer's problem: 101qs Coffee Traveler Van de Walle's Big Ideas are given on the previous page	Activity 14.14 Making "to-go" boxes (page 316 – Van de Walle) Surface Area of Right Prisms & Cylinders 7/8 unit	Activity 14.15 What is the ratio of diameter to circumference? (page 316 – Van de Walle) Figures 14.15&14.16 Development of the formula for area of a circle (page 3.18 – Van de Walle) TIPS4RM - Unit 7 - gr 8 TIPS4RM - Unit 3 - gr 8 NCTM Applet Illuminations - Circle Tool Would you Rather… - 3/4 of a circle or the outer portion? - Travel along the triangles or the circumference?

How can using dynamic geometric software (e.g., Geometer's Sketchpad or GeoGebra) help our students visualize and conceptualize measurement concepts?

Students who develop problem-solving skills that enable them to explore extensions to measurement problems have the following learning characteristics:

- They have a rich understanding of measurement units and estimation strategies.
- They can pose "what if" questions to extend problems in new mathematical directions.
- They are willing to persevere in their mathematical thinking and solve mathematical problems.
- They can work collaboratively with others.

Measurement Verbs in the Ontario Mathematics Curriculum:
What experiences are we providing our students in Measurement?

Primary

Junior

Inter-mediate

Sometimes Always Never

Try asking your students to consider if they could make a sometimes / always / never statement in Measurement (and other strands as well!)

Primary	Junior	Intermediate
Sometimes all four sides of a rectangle are the same length	Sometimes a rectangle is a square and sometimes a rhombus is a square	Sometimes the diagonals of a quadrilateral are equal in length
There are always 100 centimetres in a metre	You can always represent a multiplication with an array / rectangle	The ratio of circumference to diameter of any circle is always pi – π (3.14)
There are never 100 minutes in an hour	You can never go to the left on a clock to figure out elapsed time because time never goes backward	If the radius of a circle is a whole number, the area will never be a whole number

Assessment Idea